NONSTATIONARY BURNING OF PROPELLANTS WITH VARIABLE SURFACE TEMPERATURE

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The author examines nonstationary processes (combustion at varying pressure, quenching, and ignition) for a model propellant whose burning rate u and surface temperature T_1 depend on pressure p and initial temperature T_0. All the processes in the surface reaction zone and the gas phase are assumed inertialess. It is shown that a theory of nonstationary combustion for such a model can be constructed by analogy with the Zel'dovich theory [1, 2], in which the surface temperature of the powder is assumed fixed. The variation of burning rate with time has been investigated for small sudden pressure changes. It is shown how a sufficiently large and steep pressure drop may cause quenching of the propellant. The process of propellant ignition is subjected to a qualitative analysis.

1. Stationary and nonstationary laws of combustion. Under stationary conditions the burning rate and surface temperature of a propellant powder depend on the initial temperature and pressure

$$u = u^\circ (T_0, p), \qquad T_1 = T^\circ{}_1 (T_0, p). \qquad (1.1)$$

Zel'dovich [1, 2] has proposed a method of investigating nonstationary combustion processes for the case of constant surface temperature. Essentially,

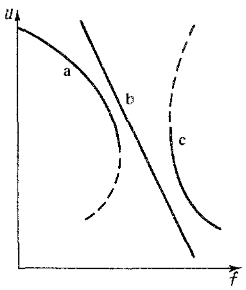

Fig. 1

the method consists of transforming the stationary dependence of burning rate $u^\circ (T_0, p)$ on initial temperature and pressure into the relation u(f, p), where f is the temperature gradient at the surface of the propellant. The relation obtained is also valid under nonstationary conditions (for this reason the superscript has been dropped from the burning rate), since the gradient determines the temperature in the combustion zone, on which the burning rate also depends. The transition from $u^\circ (T_0, p)$ to u(f, p) is realized with the help of the known relation between gradient, burning rate, and initial temperature:

$$\varkappa f^\circ = u^\circ (T^\circ{}_1 - T_0), \qquad (1.2)$$

which is valid under stationary conditions ($\varkappa$ is the thermal diffusivity of the propellant). Of course, this approach to the study of nonstationary phenomena neglects the inertia of all the processes except for heat conduction in the condensed phase.

The author of [3], making the same assumption about the leading role of the inertia of the heated layer of the condensed phase, has shown that by means of (1.2) the stationary relation between surface temperature, initial temperature, and pressure can be reduced to the relation $T_1(f, p)$, which is also valid for variable pressure.

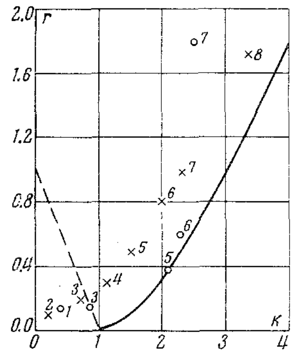

Fig. 2

Thus, even in the case of variable surface temperature the nonstationary phenomena associated with the burning of powders can be investigated by means of the nonstationary laws

$$u = u (f, p), \qquad T_1 = T_1 (f, p) \qquad (1.3)$$

obtained from the stationary laws of combustion (1.1) by using relation (1.2).

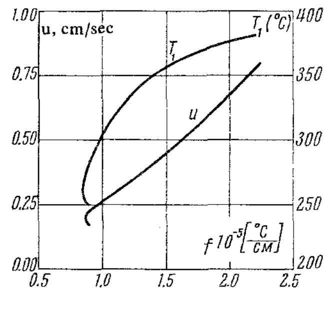

Fig. 3

The stationary laws of combustion can be obtained either from the theory of combustion with account for the specific physicochemical processes that take place in the condensed and gas phases or from experiments on the stationary burning of propellants at various pressures and initial temperatures.

Zel'dovich has shown that in the case of constant surface temperature a stable stationary combustion regime can be realized only if

$$k < 1, \qquad k = (T_1^\circ - T_0) \left(\frac{\partial \ln u^\circ}{\partial T^\circ} \right)_p . \qquad (1.4)$$

Since

$$\left(\frac{\partial \ln u}{\partial \ln f}\right)_p = \frac{k}{k-1},$$

it is obvious that the propellant can burn only on those parts of the $u(f)$ curve on which the derivative du/df is negative. In particular, in the case of an exponential relation between burning rate and initial temperature $u^\circ \sim \exp \beta T_0$ (this relation follows from the Arrhenius law for the rate of the chemical reaction in gas phase) the $u(f)$ curve at constant pressure has the form shown in Fig. 1 (curve a). The part of the curve depicted by a dashed line corresponds to unstable combustion regimes $k > 1$, i.e., to low initial temperatures, since $k = \beta(T_1^\circ - T_0)$.

The analysis of experimental data on the stationary burning of propellants often leads to a law $u^\circ(T_0)$ that is nonexponential. In this case, of course, the form of the $u(f)$ curve also changes. Thus, for example, the interpolation of $u^\circ(T_0)$ in the form $u^\circ \sim (1 - \beta T_0)^{-1}$ used in [4] gives a linear relation between burning rate and gradient (curve b). It is also possible to visualize a $u(f)$ relation opposite to case a. If the burning rate is interpolated by means of the law $u^\circ \sim \exp \beta T_0^2$, then at small values of the initial temperature the rate will decrease with increase in gradient, while at large values it will increase (curve c). However, for any relation between burning rate and gradient, stable regimes correspond to those parts of the $u(f)$ curves on which $du/df < 0$ (these are depicted by a solid line).

The situation is different if the surface temperature of the propellent itself changes with the initial temperature. In this case the criterion of stability of the stationary combustion regime at constant pressure has the form [3]

$$r > (k-1)^2/(k+1), \qquad r = (\partial T_1^\circ / \partial T_0)_p, \qquad (1.5)$$

where k has it former significance, and r is the derivative of the surface temperature with respect to the initial temperature measured in the stationary regime.

Correct to experimental errors, the data for N powder, the only system for which measurements have so far been made [5], satisfy criterion (1.5). Figure 2 presents the curve $r = (k - 1)^2/(k + 1)$ and the points corresponding to combustion regimes at various pressures and initial temperatures. The crosses correspond to a pressure p = 20 atm, the circles to p = 1 atm. The figures 1–8 correspond to initial temperatures of −200° C, −150° C, −100° C, 0° C, 100° C and 140° C. As the initial temperature increases, so do the parameters k and r.

We now turn to the relation $u(f)$. As in the case of constant surface temperature its nature will be determined by the specific stationary laws $u^\circ(T_0)$ and $T_1(T_0)$. However, stable stationary regimes correspond to parts of the $u(f)$ curve with both negative and positive values of the derivative du/df. Indeed, as shown in [3], for variable surface temperature we have

$$\left(\frac{\partial \ln u}{\partial \ln f}\right)_p = \frac{k}{k+r-1},$$

$$\frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1}{\partial \ln f}\right)_p = \frac{r}{k+r-1};$$

therefore, the sign of the derivatives du/df and dT_1/df is determined by the sign of $k + r - 1$. In Fig. 2 the dashed line represents the straight line $r = 1 - k$. It is clear from the drawing that in stable regimes, i.e., when condition (1.5) is satisfied, there may be cases of both positive and negative derivatives of the burning rate and surface temperature with respect to the gradient. Figure 3 shows $u(f)$ and $T_1(f)$ at a pressure of p = 20 atm for N powder.

Variability of the surface temperature leads to a series of important effects lacking in the constant-temperature model: the region of stable combustion increases [3]; the powder has a natural frequency, so that the dependence of the amplitude of the burning rate on the frequency of the applied, harmonically varying pressure is of the resonance type; finally, nonlinear undamped oscillations of the burning rate are possible at constant pressure [6, 7]. It is natural to expect that the behavior of the propellent at varying pressure, i.e., nonstationary combustion, will also be different from the case of constant surface temperature considered by Zel'dovich. Below we investigate certain effects associated with nonstationary combustion in the presence of a variable propellent surface temperature.

2. Small variations of pressure. We will consider the dependence of burning rate on time as the pressure varies from a certain initial value p° to an end value $p_1 = p^\circ(1 + h)$. The inertia of the condensed phase ($x < 0$), taken into account in the heat conduction equation

$$\frac{\partial T}{\partial t} = \varkappa \frac{\partial^2 T}{\partial x^2} - u\frac{\partial T}{\partial x},$$

$$T(-\infty) = T_0, \qquad T(0) = T_1, \qquad (2.1)$$

means that the burning rate u(t) does not correspond to its stationary value u°(p) at the instantaneous value of the pressure p(t). For a complete formulation of the problem, apart from (2.1), it is also necessary to assign the nonstationary laws of combustion $u(f, p)$ and $T_1(p)$ and the relation between pressure and time p(t). We will start by going over to dimensionless variables:

$$\theta = \frac{T - T_0}{T_1^\circ - T_0}, \qquad \xi = \frac{u^\circ(p^\circ)}{\varkappa}x,$$

$$\tau = \frac{[u^\circ(p^\circ)]^2}{\varkappa}t \qquad \vartheta = \frac{T_1 - T_0}{T_1^\circ - T_0},$$

$$v = \frac{u}{u^\circ(p^\circ)}, \qquad \eta = \frac{p}{p^\circ}, \qquad \varphi = \frac{f}{f^\circ}. \qquad (2.2)$$

In these variables the problem is formulated as follows: To find the dependence of burning rate on time for a given variation of pressure with time, when the burning rate and surface temperature are related in a certain way with gradient and pressure

$$v = v(\varphi, \eta), \quad \vartheta = \vartheta(\varphi, \eta), \qquad (2.3)$$

and the temperature inside the propellent satisfies the heat conduction equation

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\xi^2} - v\frac{\partial\theta}{\partial\xi}, \qquad \theta(-\infty) = 0, \qquad \theta(0) = \vartheta. \qquad (2.4)$$

Since the heat conduction equation is nonlinear, the exact solution of the problem for arbitrary relations (2.3) and $\eta(\tau)$ encounters considerable mathematical difficulties. Accordingly, we will first investigate the case of small variations of pressure, i.e., the linear approximation.

We represent the temperature, the burning rate, gradient, and pressure in the form

$$\theta = e^\xi(1 + \theta_1), \qquad \vartheta = 1 + \vartheta_1,$$

$$v = 1 + v_1, \ \varphi = 1 + \varphi_1, \ \eta = 1 + \eta_1, \qquad (2.5)$$

where

$$\varphi_1 = \vartheta_1 + \left.\frac{\partial\theta_1}{\partial\xi}\right|_{\xi=0}.$$

Linearizing the heat conduction equation, we obtain

$$\frac{\partial\theta_1}{\partial\tau} = \frac{\partial^2\theta_1}{\partial\xi^2} + \frac{\partial\theta_1}{\partial\xi} - v_1,$$

$$\theta_1(-\infty) = 0, \qquad \theta_1(0) = \vartheta_1 \qquad (2.6)$$

In accordance with [6], relations (2.3) are written in the form

$$v_1=\frac{k}{k+r-1}\varphi_1+\frac{\delta-\nu}{k+r-1}\eta_1,$$
$$\vartheta_1=\frac{r}{k+r-1}\varphi_1-\frac{\delta+\mu}{k+r-1}\eta_1\,. \tag{2.7}$$

Here, k and r are determined by means of (1.4) and (1.5); ν and μ are parameters characterizing the pressure dependence of the burning rate and surface temperature:

$$\nu=\left(\frac{\partial\ln u^\circ}{\partial\ln p}\right)_{T_0},\qquad \mu=\frac{1}{T_1^\circ-T_0}\left(\frac{\partial T_1^\circ}{\partial\ln p}\right)_{T_0}. \tag{2.8}$$

Finally,

$$\delta=\nu r-\mu k\,. \tag{2.9}$$

In order to solve the problem we used the Laplace transformation

$$F(p)=p\int_0^\infty e^{-p\tau}F(\tau)\,d\tau\,.$$

From (2.6) we have

$$p\theta_1(p)=\theta_1''(p)+\theta_1'(p)-v_1(p),$$
$$\theta_1(p)|_{\xi=-\infty}=0,\quad \theta_1(p)|_{\xi=0}=\vartheta_1(p),$$

where a prime denotes differentiation with respect to ξ. The solution of this equation with account for the boundary conditions is

$$\theta_1(p)=[\vartheta_1(p)+v_1(p)/p]\,e^{z\xi}-v_1(p)/p,$$
$$z=-{}^1\!/_2+\sqrt{p+{}^1\!/_4}\,\cdot \tag{2.10}$$

Hence the Laplace-transformed correction to the gradient

$$\varphi_1(p)=\vartheta_1(p)+z\,[\vartheta_1(p)+v_1(p)/p]\,. \tag{2.11}$$

Transforming (2.7), we obtain two more equations for determining $v_1(p)$, $\vartheta_1(p)$ and $\varphi_1(p)$

$$v_1(p)=\frac{k}{k+r-1}\varphi_1(p)+\frac{\delta-\nu}{k+r-1}\eta_1(p)$$
$$\vartheta_1(p)=\frac{r}{k+r-1}\varphi_1(p)-\frac{\delta+\mu}{k+r-1}\eta_1(p)\,. \tag{2.12}$$

From the last three equations we find that

$$v_1(p)=\frac{\nu+\delta z}{1-k+(r+k/p)z}\eta_1(p),$$
$$\vartheta_1(p)=\frac{\mu+\delta(1-z/p)}{1-k+(r+k/p)z}\eta_1(p)$$
$$\varphi_1(p)=\frac{\nu z/p+\mu(1+z)+\delta(1+z-z/p)}{1-k+(r+k/p)z}\eta_1(p). \tag{2.13}$$

When $p \ll 1$ these expressions correspond to a very slow variation of pressure (quasi-stationary regime). Confining ourselves to the first term of the expansion in p, for the burning rate we have

$$v_1(p)=\nu\eta_1(p)+k(\nu-\mu)\,p\eta_1(p)\,.$$

The primitive of the first term is simply the correction to the burning rate for variation of pressure under stationary conditions $\nu\eta_1(\tau)$. The second term, however, is proportional to the derivative of the pressure with respect to time (η_1 at $\tau = 0$ is equal to zero).

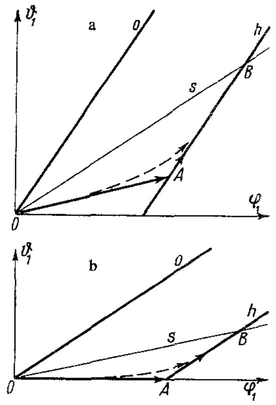

Fig. 4

Thus, in the quasi-stationary regime we have

$$v_1(\tau)=\nu\eta_1(\tau)+k(\nu-\mu)\;d\eta_1/d\tau,$$
$$\vartheta_1(\tau)=\mu\eta_1(\tau)+r(\mu-\nu)d\eta_1/d\tau,$$
$$\varphi_1(\tau)=(\nu+\mu)\eta_1(\tau)+(\nu-\mu)(k+r-1)d\eta_1/d\tau, \tag{2.14}$$

i.e., for slow variation of pressure the burning rate, the surface temperature, and the gradient differ from their stationary values at the instantaneous value of the pressure $\eta_1(\tau)$ by amounts proportional to the rate of change of pressure. This result was obtained by Zel'dovich in [2] for a model with constant surface temperature. At $r = \mu = 0$ expression (2.14) for the burning rate goes over into the relation obtained in the above-mentioned study.

We will now consider the case of a sharp change in pressure. Let the pressure change from $\eta_1 = 0$ to $\eta_1 = h$, at time $\tau = 0$, and thereafter remain constant.

To some extent this is an abstract formulation of the problem. Firstly, in reality it is not possible to bring about a sudden rise or fall in pressure, and, secondly, in constructing the propellent model investigated we assume that the relaxation times of the processes taking place in the gas phase and the surface zone are equal to zero. Actually, they are nonzero. However, an examination of the nonstationary phenomena using this simple relation between pressure and time makes it possible to clarify a number of important points connected with the variation of burning rate, surface temperature, and gradient and then to pass to the investigation of an actual case of pressure variation at a finite rate.

Small times correspond to large values of the Laplace variable. Setting $p \gg 1$ and $\eta_1(p) = h$, from (2.13) we obtain

$$v_1(p)=\frac{\delta}{r}\left[1+\frac{k(\mu+\delta)}{r\delta\sqrt{p}}\right]h,$$
$$\vartheta_1(p)=\frac{\mu+\delta}{r\sqrt{p}}h,$$
$$\varphi_1(p)=\frac{\mu+\delta}{r}\left[1+\frac{k+r-1}{r\sqrt{p}}\right]h.$$

Consequently,

$$v_1(\tau)=\frac{\delta}{r}\left[1+\frac{2k(\mu+\delta)}{r\delta}\left(\frac{\tau}{\pi}\right)^{1/2}\right]h,$$

$$\vartheta_1(\tau) = \frac{2(\mu+\delta)}{r}\left(\frac{\tau}{\pi}\right)^{1/2} h,$$

$$\varphi_1(\tau) = \frac{\mu+\delta}{r}\left[1 + \frac{2(k+r-1)}{r}\left(\frac{\tau}{\pi}\right)^{1/2}\right] h. \qquad (2.15)$$

In Fig. 4a and b the straight lines o and h correspond to relations (2.7) for $\eta_1 = 0$ and $\eta_1 = h$, respectively. The initial state of the propellant is represented by the point O, the end state by the point B.

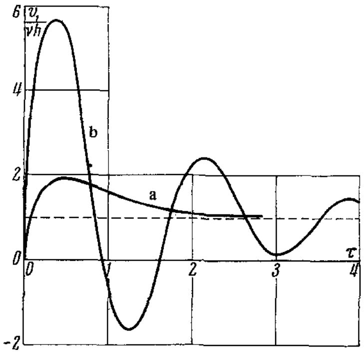

Fig. 5

Stationary combustion regimes correspond to the straight lines s, whose equations are

$$v_1 = \frac{\varphi_1}{1+\mu/\nu}, \qquad \vartheta_1 = \frac{\varphi_1}{1+\nu/\mu}.$$

These expressions were obtained from the laws of stationary combustion $v_1 = \nu\eta_1$ and $\vartheta_1 = \mu\eta_1$ with account for relations (2.7). The coordinates of the point B are

$$v_1(B) = \nu h, \qquad \vartheta_1(B) = \mu h, \qquad \varphi_1(B) = (\mu+\nu)h.$$

For a sharp change of pressure, as follows from (2.15), the surface temperature remains constant, but the burning rate gradient changes abruptly from zero to

$$v_1(A) = \frac{\delta}{r} h, \qquad \varphi_1(A) = \frac{\mu+\delta}{r} h.$$

Graphically, the change in the state of the propellents at time $\tau = 0$ is represented by the arrows OA.

From the point A the system begins to move on the straight line AB in the direction of the end state, the point B. Actually, the coefficients of $\sqrt{\tau}$ in (2.15) are positive; it is clear from the graph that $k + r - 1 > 0$ (the slope of the straight lines o and h is positive) and $\mu + \delta > 0$ (at a given gradient the surface temperature decreases with increase in pressure). When the pressure changes at a large, but finite rate, the change in the state of the system at small τ is depicted by the dashed curve.

Physically it is easy to understand why when the pressure changes sharply the surface temperature changes only slightly, and the burning rate and gradient strongly. As a result of the thermal inertia of the surface reaction zone the temperature profile cannot change significantly when the pressure rises rapidly. However, a small increase in the surface temperature of the propellant is enough to cause a sharp increase in the gradient and hence the burning rate.

We note that in the constant surface temperature model the gradient remains constant when the pressure suddenly increases, while the burning rate takes a value

$$v_1(A) = \frac{\nu}{1-k} h$$

greater than its end value $v_1(B) = \nu h$. There then begins a gradual decrease in burning rate to the value $v_1(B)$. In the case of variable surface temperature at the first instant all the quantities (burning rate, temperature, and gradient) take values smaller than at point B.

We will now consider how the stationary end regime is approached. For this purpose we take expression (2.13) for the burning rate and, using the rules of operational calculus [9], we find the inverse transform $v_1(\tau)$. Calculations lead to the following result:

$$\frac{v_1(\tau)}{h} = \left(\frac{\delta}{r} - \frac{\nu}{2}\right)\left[2e^{-\lambda\tau}\cos\omega\tau - e^{-1/4\tau}\, U\left(\frac{\omega r}{k-1}\sqrt{\tau}, \frac{k-1}{2r}\sqrt{\tau}\right)\right] + \frac{k-1}{2\omega r^2}\left(\frac{k(\delta+\mu)}{r} + \frac{\nu(1-r+k)}{2}\right) \times \left[2e^{-\lambda\tau}\sin\omega\tau + e^{-1/4\tau}\, V\left(\frac{\omega r}{k-1}\sqrt{\tau}, \frac{k-1}{2r}\sqrt{\tau}\right)\right] + \frac{\nu}{2}\operatorname{erfc}\left(-\frac{\sqrt{\tau}}{2}\right). \qquad (2.16)$$

$$\omega = \sqrt{\omega_0^2 - \lambda^2}, \qquad \omega_0 = \frac{\sqrt{k}}{r},$$
$$\lambda = \frac{r(k+1) - (k-1)^2}{2r^2}. \qquad (2.17)$$

Here, ω_0 and λ are the natural frequency and logarithmic decrement of the oscillations of the burning rate introduced in [6]. The functions U(x, y) and V(x, y)

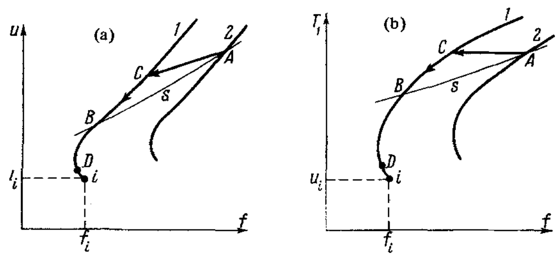

Fig. 6

are related with the error function of complex argument by the simple relations

$$U(x, y) + iV(x, y) = W(z), \qquad z = x + iy,$$

where

$$W(z) = e^{-z^2}\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2} dt\right].$$

The function W(z) has been tabulated in [10]. Asymptotically at $\tau \gg 1$

$$\frac{v_1(\tau)}{\nu h} = 1 + \frac{2e^{-\tau/4}}{\sqrt{\pi\tau}} + 2e^{-\lambda\tau}\left[\left(\frac{\delta}{r\nu} - \frac{1}{2}\right)\cos\omega\tau + \frac{k-1}{2r^2\omega\nu}\left(\frac{k(\delta+\mu)}{r} + \frac{\nu(1-r+k)}{2}\right)\sin\omega\tau\right]. \qquad (2.18)$$

To a large extent the nature of the relation $v_1(\tau)$ depends on the value of λ. If λ is large, the oscillating term damps rapidly. Otherwise, at small λ the oscillations of the burning rate continue for a long time.

In this case the variation of surface temperature and gradient is also oscillatory in character.

Figure 5 presents typical relations between burning rate and time.

Curve a was constructed for a propellent with the following parameters:

$$k = 1.5, \quad r = 0.5, \quad \nu = {}^2/_3, \quad \mu = {}^1/_6.$$

In this case

$$\omega_0 = \sqrt{6}, \quad \lambda = 2, \quad \omega = \sqrt{2}.$$

These parameters correspond to N powder burning at a pressure of 20 atm with an initial temperature of 20° C. Curve b was constructed for a propellant with parameters selected so that λ was small as compared with the frequency of the oscillations. In this case

$$k = 2, \ r = 0.4, \ \nu = {}^2/_3 \ \mu = 0,$$

$$\omega_0 = 2.5\ \sqrt{2}, \ \lambda = 0.625, \ \omega \approx 3.5.$$

In both cases the nonstationary burning rate is significantly higher than its end value $v_1(\infty) = \nu h$. Curve a has a single maximum. In the case of oscillations, however, the value νh is passed repeatedly, and the rate may even fall below its initial value.

3. Large variations of pressure. Quenching of the propellant. For large changes in pressure the linear approximation is no longer sufficient. In this case it is necessary to solve the nonlinear heat conduction equation (2.4) with allowance for the nonlinear relations between burning rate and surface temperature and gradient and pressure (2.3). Of course, the qualitative nature of the variation of burning rate in time will be the same as in the case of small pressure changes examined above. In particular, it remains true that the temperature changes only slightly, whereas the burning rate and gradient change abruptly in the initial moments.

Obviously, the asymptotic behavior of the burning rate at large times will be characterized by a component oscillating with frequency ω. However, at large amplitudes of the burning rate oscillations, undamped and even growing nonlinear oscillations may occur.

This is connected with the fact that when the nonlinear properties of the system are taken into account λ begins to depend on the amplitude of the oscillations [7]. The detailed behavior of the burning rate for a given change of pressure can be obtained either by numerical integration of the starting equations or by some approximate method, for example, the method of integral relations used in conjuction with the constant-temperature model in [4, 8, 11, 12].

Zel'dovich has studied the problem of the quenching of a propellant with rapid decrease in pressure. This effect is associated with the fact that the temperature gradient at the surface of the propellant in the stationary regime is the greater, the higher the pressure. On the other hand, assuming $u^\circ \sim \exp \beta T_0$ there is a maximum on the $f(u)$ curve at any pressure (see Fig. 1, curve a). If after the pressure drops the gradient exceeds its maximum value at the end pressure, combustion is impossible and the powder is quenched.

Thus, the form of the $u(f)$ curve is important in explaining quenching in the constant-temperature model. In fact, quenching can be explained only in the case of curve a. For the $u(f)$ relations represented by curves b and c (Fig. 1) quenching is impossible.

We will now consider how to explain quenching when the pressure falls quite rapidly and steeply in the case of a propellant whose surface temperature is variable. In our opinion, for a correct understanding of this phenomenon it is necessary to know the behavior of the $u(f)$ and $T_1(f)$ curves at sufficiently low u and T_1. Figure 3 was constructed using experimental data relating to stationary combustion in the range of initial temperatures $-200°\ C \le T_0 \le +140°\ C$. What results are

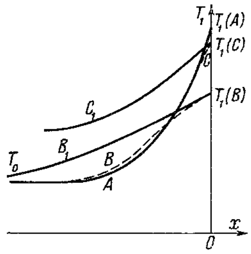

Fig. 7

to be expected with further decrease in the initial temperature of the propellent? It may be that stationary combustion becomes impossible at a certain temperature. In this case the $u(f)$ and $T_1(f)$ curves end at the points (f_i, u_i) and (f_i, T_{1i}). Another variant is also possible—stationary combustion exists down to an initial temperature equal to absolute zero. Then the nonstationary laws $u(f)$ and $T_1(f)$ can be obtained from the stationary relations $u^\circ(T_0)$ and $T_1^\circ(T_0)$ only up to certain values of the burning rate and surface temperature. However, the nonstationary relations are also meaningful below those values. Their determination in that region must be based on experiments with nonstationary combustion (for example, in experiments on propellant ignition). It is natural to expect that in this case, too, combustion will take place only if the surface temperature is higher than a certain value, i.e., the curve ends at a certain point (f_i, T_{1i}). This point corresponds to the end point of the burning rate curve (f_i, u_i).

Figures 6a and b show the burning rate and surface temperature as a function of the gradient for two values of the pressure $p_1 < p_2$ (curves 1 and 2, respectively). The curves s correspond to stationary combustion regimes at the given propellant temperature T_0 and various pressures. We will consider the qualitative behavior of the propellent as the pressure varies from p_2 to p_1. The initial stage is represented by the point A, the end state by point B. For a slow variation of pressure the transient process is represented by the curve s. In the case of a rapid change of pressure the surface temperature changes only slightly (in the limiting case of a sudden drop in pressure it does not change at all), while the gradient and burning rate experience sharp changes. This is shown by the arrows AC. There is almost no change in the temperature profile inside the propellant as the pressure varies. In Fig. 7 the solid curve A corresponds to the initial temperature distribution, and the dashed curve C, which differs slightly from A only close to the surface of the propellant, represents the temperature profile immediately after the fall in pressure—the surface temperature has changed slightly from $T_1(A)$ to $T_1(C)$, but the gradient at the surface has changed sharply. The new value of the gradient corresponds to the stationary temperature profile C_1, which in turn corresponds to a large initial temperature. If the pressure subsequently remains constant and equal to p_1, the state of the propellant will vary along curve 1 (see Fig. 6) in the direction CB. The surface temperature and burning rate will fall. In fact, for the temperature profiles C and C_1 near the surface of the propellant we have the relations

$$\left(\frac{\partial T}{\partial t}\right)_{C_1} = 0, \qquad \left(u\,\frac{\partial T}{\partial x}\right)_C = \left(u\,\frac{\partial T}{\partial x}\right)_{C_1},$$

$$\left(\varkappa\,\frac{\partial^2 T}{\partial x^2}\right)_C < \left(\varkappa\,\frac{\partial^2 T}{\partial x^2}\right)_{C_1}.$$

and, consequently, from heat conduction equation (2.1)

$$(\partial T\,/\,\partial t)_C < 0.$$

If the drop in pressure was sufficiently sharp, then on approaching the point B the temperature profile in the propellant will be sharply different from the stationary profile (curves B and B_1 in Fig. 7), which for the same reasons leads to a further decrease in the burning rate and surface temperature, i.e., to the displacement of point B in the direction of the end point of the curves i.

The subsequent progress of the nonstationary process depends on the magnitude and steepness of the pressure drop. For a small value of p_2/p_1 or a slow variation of pressure on the segment Bi we get a temperature profile for which the second derivative of the temperature near the surface coincides with the second derivative of the stationary profile. In this case at point D the fall in temperature ceases, and an oscillatory approach to the stationary regime begins. However, for large and sharp pressure drops the nonstationary process may reach the point i, at which combustion ceases—the propellant is quenched.

Thus, the basic reason for quenching of the propellent in the variable-temperature model is the same as in the Zel'dovich model—the sharp difference between the temperature profiles in the initial and end states. However, the detailed behavior of the burning rate and temperature distribution and the quenching criterion are significantly different. At constant surface temperature quenching is determined by the point with an infinite derivative on the $u(f)$ curve; in the variable-temperature model by the point i corresponding to the end of the $u(f)$ and $T_1(f)$ curves. In the absence of quenching, relaxation of the temperature profile to the stationary distribution is "viscous" in the first case and oscillatory in the second.

We note that both models give qualitatively identical relations between the minimum drop and rate of variation of pressure necessary for quenching. With increase in the ratio p_2/p_1 the minimum rate of fall of pressure sufficient for quenching decreases. Actually, the occurrence of quenching depends on the distance between the point C and the point B, and this distance increases with increase in the magnitude and rate of the pressure drop. Thus, the quenching curves, i.e., the dependence of the ratio p_2/p_1 sufficient for quenching on the rate of fall of pressure dp/dt, will have qualitatively the same form as in the constant-temperature model (the quenching curves for that case were approximately calculated in [11]). In this connection, we note that the experimental data (e.g., [13]) are qualitatively explained by both models. To make a quantitative analysis of the quenching phenomenon and compare the results with the experimental data it is necessary, of course, to know the detailed form of the $u(f, p)$ and $T_1(f, p)$ relations. Hence the important, in our opinion, experimental problem of determining the stationary relations $u^\circ(T_0, p)$ and $T_1^\circ(T_0, p)$ over broader intervals of variation of pressure and initial temperature than before.

4. **Propellent ignition.** As Zel'dovich has shown, at constant surface temperature to ignite a propellant it is necessary to heat its surface to a certain temperature and create a sufficient reserve of heat in the condensed phase, so that the temperature gradient is less than that maximally possible at the given pressure and the state of the propellant is described by a point lying on a segment of the $u(f)$ curve corresponding to stable combustion regimes. Different propellant ignition regimes depending on the rate of heat supply, have been examined by Librovich [8], who has shown that at a sufficiently intense rate of heat supply ignition does not occur at all—gasification of the propellants is accompanied by the establishment of a stationary regime with a temperature gradient at which a flame cannot exist above the surface.

We now turn to the case of variable surface temperature. Obviously, ignition can occur after the surface temperature reaches the value T_{1i}. Before heating begins, $T_1 = T_0$ and $f = 0$. As the propellant is heated, both the surface temperature and the gradient begin to increase. By the same method as employed in studying the quenching of a propellant it is easy to show that if the surface temperature reaches the value T_{1i}, while at that instant the gradient f_0 is greater than f_i, then after ignition the surface temperature must fall, i.e., the propellant must be quenched. In fact, after ignition the nonstationary relation $T_1(f)$ requires a single-valued relation between surface temperature and gradient. Therefore at the instant of ignition the temperature profile must change so that the gradient decreases from f_0 to f_i. However, in the presence of the resulting temperature distribution the surface temperature will decrease (the second derivative of the temperature is less than in the stationary profile corresponding to the gradient f_i). Thus, rapid heating of the propellant may lead only to flashing.

To ignite the propellant it is necessary to heat it sufficiently slowly, so that at the moment when the surface temperature reaches the value T_{1i} the gradient at the surface is not greater than f_i. Only then will the surface temperature rise after ignition and the burning rate increase with subsequent relaxation to the stationary regime (in this case oscillations about the stationary end regime are possible).

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